DEPARTMENT OF PHYSICS, UIO

FYS3610-SPACE PHYSICS

MID-TERM EXAMINATION

Date: 5 October 2009

Time of day: 15:00-1800

Permitted aid(s): Calculating machine.

The exam set consists of 3 pages, with 3 problems.

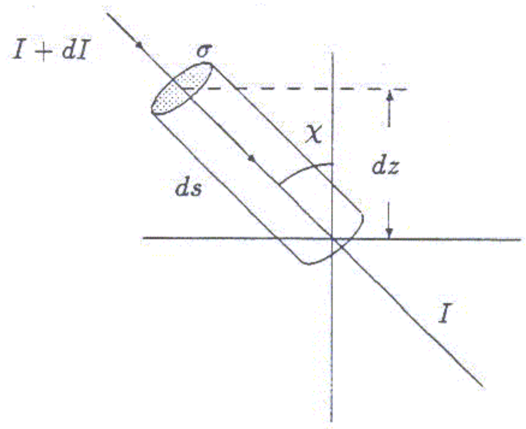
PROBLEM 1

a) Draw a sketch of the height variation of temperature in the Earth's atmosphere from sea level up to 120 km and explain in general terms what are the physical mechanisms responsible for this structure. Annotate subdivision of regions in "pauses and spheres"

b) Derive the barometric equation for an isothermal atmosphere and explain the physical meaning of the term scale height.

c) What do we mean by an adiabatic lapse rate? What is a typical value for it?

d) Assume monochromatic light at a sloping incidence with a horizontally stratified atmosphere as shown in the figure below.



Set up an expression for the absorbed radiation in an element ds of this radiation path. Define the parameters involved. Show that the intensity of incoming radiation varies with solar zenith angle as: $I = I_\infty e^{-\tau \sec\chi}$. Define τ and explain the meaning of this parameter.

e) The ion production rate for an exponential atmosphere can be written as:

$$q(\chi, z) = \frac{I_\infty \eta}{H} \tau \cdot e^{-\tau \sec(\chi)}$$

Show that the maximum production rate can be expressed as:

$$q_m(\chi, z_m) = \frac{I_\infty \eta \cos\chi}{eH} = q_m(0, z_{m0}) \cos\chi$$

Sketch a graph demonstrating how maximum ion production varies with zenith angle.

PROBLEM 2

a) Draw a sketch of the undisturbed Earth magnetic field using the Earth's rotational axis as a reference. Indicate direction of the magnetic field. What is the strength of the magnetic field near the equator and near the poles?

b) The Earth magnetic field are normally referred to the local coordinate systems (X,Y,Z) or (H, D, Z). Draw a figure that illustrates the Earth magnetic field vector decomposed in the two coordinate systems. Indicate geographic north in your figure.

c) Describe what happens when the solar wind, consisting of electrons and protons, hits the Earth magnetic field.

d) What is a typical value for the solar wind *stand-off distance*? What are the controlling parameters for this distance; i.e. to push it inwards and outwards?

e) Describe the magnetic field of the Sun.

f) Describe the *frozen-in-field* concept?

PROBLEM 3

a) Assume a static uniform electric field along the y-axis and a static uniform magnetic field along the z-axis. Draw a sketch showing the particle trajectories separately for ions and electrons. Assume the particles are initially at rest.

b) Assume a magnetic field along positive z-direction, increasing in strength along positive y. There is no electric field. Draw a sketch showing the particle trajectories separately for ions and electrons. The particles have an initial velocity along negative y. What do we call this drift?

c) Which of the two cases, a) or b), gives rise to current? Justify your answer.

d) Height integrated currents in the ionosphere can be expressed as:

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \Sigma_P & -\Sigma_H \\ \Sigma_H & \Sigma_P \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Describe the parameters involved. Give a physical explanation for the upper and lower limit of the ionospheric conductive layer.

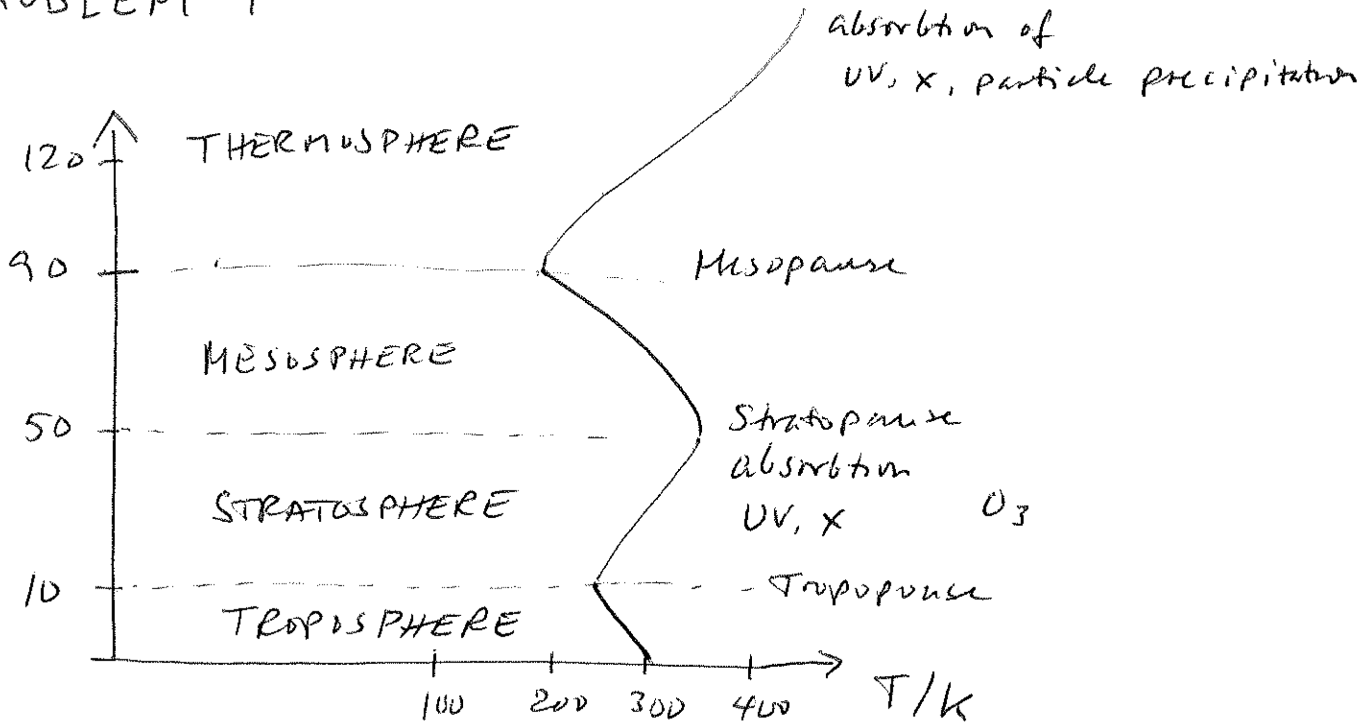
e) Assume an east-west extended arc, that Σ_H and Σ_P are both zero outside the arc, that there is no field aligned current, and that E_y is the same inside and outside the arc. Prove that the current along the arc then is given as

$$J_y^A = \left[\frac{\left(\Sigma_H^A\right)^2}{\Sigma_P^A} + \Sigma_P^A \right] \cdot E_y$$

f) Make a sketch to illustrate the magnetic field disturbance on the ground underneath the westward electrojet. Describe how it will disturb the background magnetic field (northern hemisphere).

PROBLEM 1

a)



b) $dp = -\rho g\, dz = -nmg\, dz$

$p = nkT$

$$\frac{dp}{p} = -\frac{nmg}{nkT}\, dz$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_0^z \frac{mg}{kT}\, dz$$

$$p = p_0 e^{-\frac{mg}{kT} z} = p_0 e^{-\frac{z}{H}} \quad \text{where } H = \frac{kT}{mg}$$

pressure at $z = H$: $p = \frac{p_0}{e}$

represents a measure for how fast pressure varies with height.

c) $\left.\frac{\partial T}{\partial z}\right|_{ad} \sim \frac{1°C}{100m}$ $\left(\left.\frac{\partial T}{\partial z}\right|_{ad} = -\frac{g}{c_p} \right)$

d) $dI = -I \sigma n\, ds$

I = monochromatic ligh intensity / incoming radiation intensity

σ = absorbtion cross section

n = atmospheric density / number density

ds = path element

$$\frac{dI}{I} = -I\sigma n\, ds \qquad \cos\chi = \frac{-dz}{ds}$$

$$ds = -\frac{dz}{\cos\chi} = -\sec\chi\, dz$$

$$\frac{dI}{I} = \sigma n \sec\chi\, dz$$

$$\int_{I_\infty}^{I} \frac{dI}{I} = \sec\chi \int_\infty^{z} \sigma n\, dz = -\tau\sec\chi$$

$$\ln I \Big|_{I_\infty}^{I} = -\tau\sec\chi$$

$$\boxed{I = I_\infty e^{-\tau\sec\chi}} \quad \text{where} \quad \boxed{\tau = -\int_\infty^{z} \sigma n\, dz \;\; \text{optical depth}}$$

τ - describes transparency - how rapid radiation is absorbed

z for $\tau = 1$ is used as a measure for how fast a type of radiation is absorbed.

e) $$q(\chi, z) = \frac{I_\infty \eta}{H}\tau e^{-\tau\sec\chi}$$

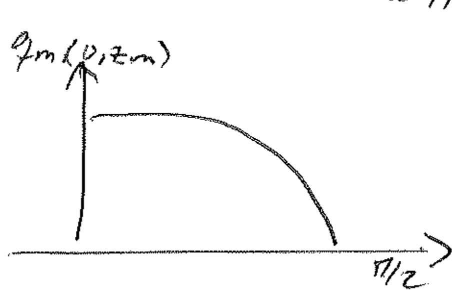
$$\frac{\partial q}{\partial z} = 0 = \cancel{\frac{I_\infty\eta}{H}}\left(\cancel{e^{-\tau\sec\chi}}\frac{d\tau}{\partial z} + \tau \cancel{e^{-\tau\sec\chi}}\left(\frac{\partial}{\partial z}(-\tau\sec\chi)\right)\right)$$

$$0 = \frac{\partial\tau}{\partial z} + \tau\left(-\sec\chi\frac{\partial\tau}{\partial z}\right)$$

$$\underline{\tau = \cos\chi}$$

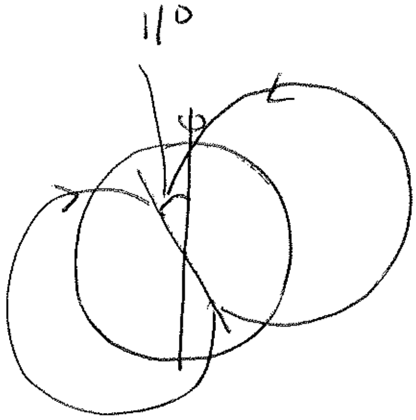
$$q_m(\chi, z_m) = \frac{I_\infty\eta}{eH}\cos\chi \qquad q_{max}(0, z_m) = \frac{I_\infty\eta}{eH}$$

$$= q_m(0, z_m)\cos\chi$$



PROBLEM 2

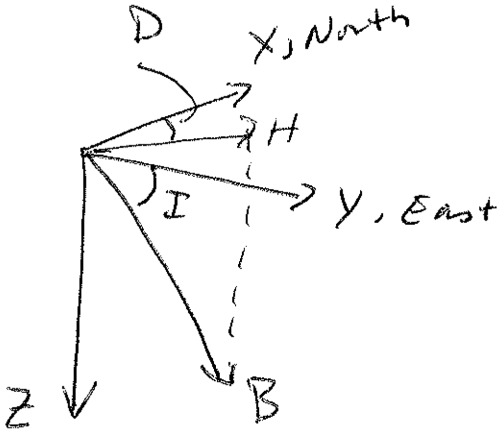
a)



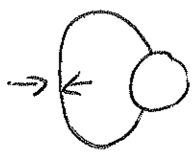
$B_{pol} = 60\,000\,nT$

$B_{eq} = 30\,000\,nT$

b)



c)

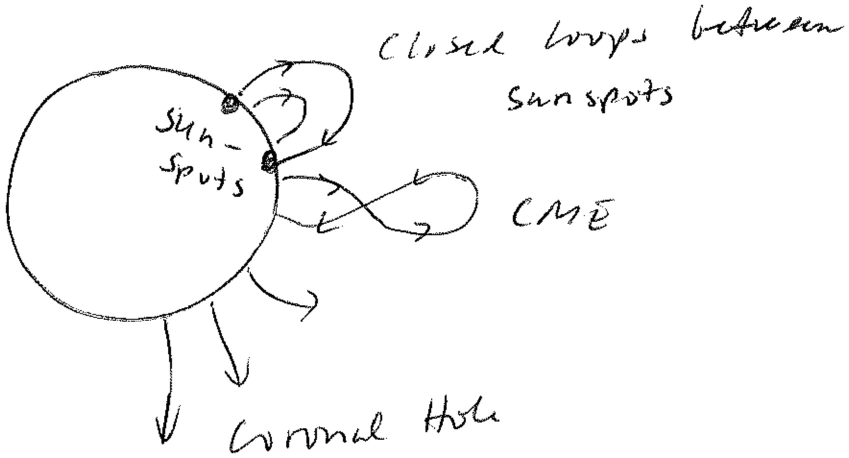


compression of the Earth magnetic field

$\underline{F} = q\,\underline{v} \times \underline{B}$

d) $r = 8-10\ R_E$

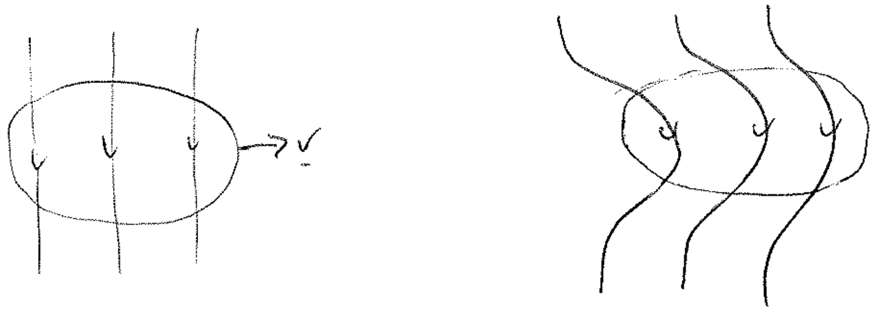
Depends on density (n) and (v) velocity in the solar wind e)



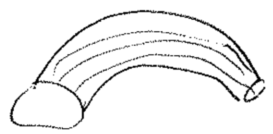
magnetic field strong near sunspots - up to 0.3T.

11-year cycle.

f)



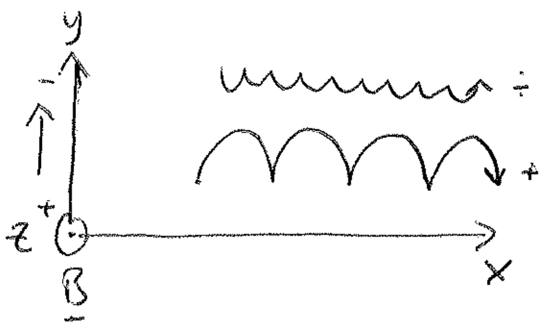
$$\frac{d\Phi}{dt} = \frac{d(BS)}{dt} = 0$$



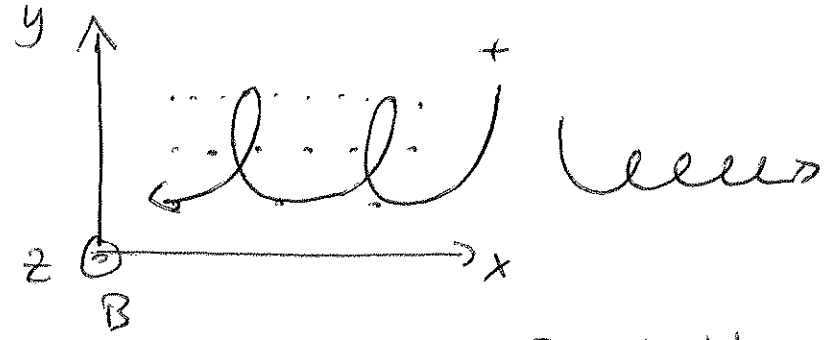
The plasma is frozen into the magnetic field. No plasma transport across the magnetic field.

PROBLEM 3

a)



b)



gradient B drift.

c) $j = ne(\underline{v_i} - \underline{v_e})$

case a) $v_i = v_e \Rightarrow$ no current ($E \times B$ - drift)

case b) $v_i \neq v_e \Rightarrow$ current

d) $J_{x,y} = \int j_{x,y}\, dz$ height integrated currents

$\Sigma_{P,H} = \int \sigma_{P,H}\, dz$ height integrated Pedersen and Hall conductivities.

$E_{x,y}$ - applied electric field in x and y directions.

upper limit: (~200 km) Frozen-in field, lack of collisions $v_i = v_e = \frac{E \times B}{B^2}$

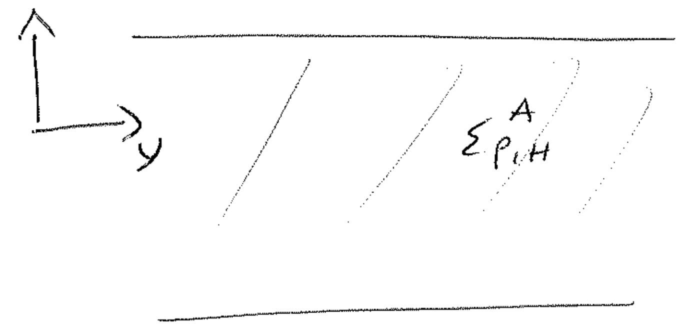
$\begin{pmatrix} \nu_{in} \ll \omega_i \\ \nu_{en} \ll \omega_e \end{pmatrix}$

lower limit: electron density drops (~90 km)

$(\sigma_{H,P} = \frac{e}{B} n_e K_{H,P} \simeq 0$

e)

$$\Sigma^0_{H,P} = 0 \ , \ J_x^0 = 0$$

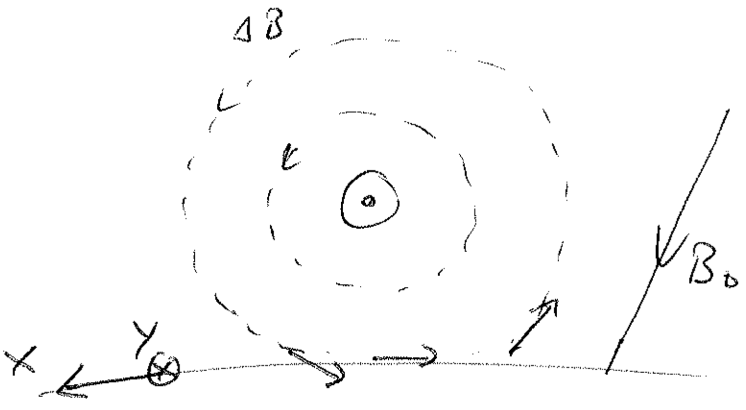


$$J_x^A = \Sigma_P^A E_x^A - \Sigma_H^A E_y = 0$$

$$\Rightarrow \quad E_x^A = \frac{\Sigma_H}{\Sigma_P} E_y$$

$$J_y^A = \Sigma_H^A E_x^A + \Sigma_P^A E_y = \left(\frac{\Sigma_H^2}{\Sigma_P} + \Sigma_P \right) E_y$$

f)



$$\Delta B_x < 0$$